Pergamon

Computers Math. Applic. Vol. 35, No. 6, pp. 137–149, 1998

0898-1221/98 $19.00 + 0.00

PII: S0898-1221(98)00025-X

An Optimal Design of Piping Route in a CAD System for Power Plant

Y. YAMADA
Electronics Division, Mitubishi Heavy Industry
1-1, Wadasaki-chou 1 Choume, Hyogo-ku, Kobe 672, Japan

Y. TERAOKA
Department of Mathematics and Information Sciences
Osaka Prefecture University
Gakuen-chou 1-1, Sakai, Osaka 593, Japan

(Received June 1996; accepted August 1996)

Abstract—The shortest path problem is one of the most important tools for the optimal design in the power plant piping CAD. This problem can be solved by modelization of spaces around the pipe lines. That is to say, those spaces are defined as the small spaces divided from the plant space and the adjacent relations among spaces are expressed with the graph representation and the shortest path problem is solved by using the power multiplication method to the graph. This paper reports an optimal design system for the piping CAD system based on the space model.

Keywords—Piping layout, Shortest path problem, Space model, Graph representation.

1. INTRODUCTION

The role of piping in a thermal or nuclear power plant is important, and the design and manufacture of the piping have a large influence on the quality and cost of the power plant. Recently, systematization has been established in almost all parts of plant piping design, due to the rapid progress of CAD, which has become indispensable for design and manufacture working. For example, composite drawings and bills of material are perfectly produced by CAD system. However, automatic design of piping routes has not yet been established, and a designer must find out the optimal route by directly observing piping routes graphically displayed on screen by a CAD system (Figure 1).

A power plant is constructed in a large building. In the case of nuclear power, the plant is constructed in a concrete building, which is divided into many rooms enclosed within concrete walls for safety and airtightness.

Since many pipelines are constructed in the divided rooms, the density of the pipelines is very high, and the rooms are very crowded. Functions required of the plant have recently increased with complicated conditions.

Under such conditions, the difficulty of designing pipelines significantly increased. As such a designer must spend much more time and have much experience to design plant piping.

The reason why a designer must depend on observations is that space situations around pipelines are not modelized for CAD systems. Thus, such systems cannot determine automatically the space for pipelines, in which automatic design is an almost impossible problem.

CAD systems presently used by plant makers generally contains data concerning shape and position of the pipelines, equipment, and building structures.

Typeset by AMS-TEX

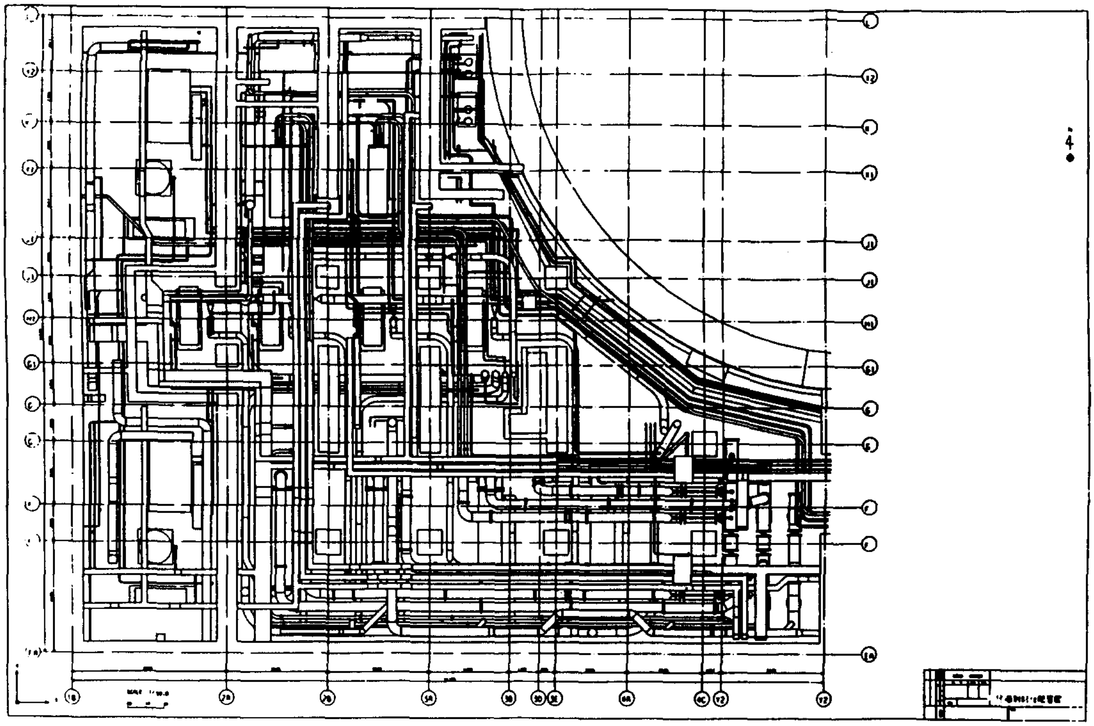


Figure 1. A sample of nuclear power plant piping.

Geographical information is mathematically represented by the computational geometry graph [1,2] and the solution of the optimal problem for post location is very famous [1]. In the IC design field, the space region for parts location and lines and connecting points is mathematically represented [3]. The optimal design for an integrated circuit is done automatically.

In the three-dimensional piping design field, there exists little research that deals with the plant space needed for the piping [4,5]. The space dealt with in such research, however, is defined only by the equipment and building structures. The spaces occupied by those elements are defined as the not-permitted regions for piping routes.

However, the influence of already installed pipelines as an obstacle region was not considered in the above papers. Therefore, we cannot expect preciseness when piping routes are automatically designed.

We can model not only pipelines data, but also data of the space around the pipelines. Thus, a total model for the piping can be constructed by using CAD. This total model is represented by the same as the above geographical representation. Various kinds of graphs for the applications are extracted from this total graph representation.

We assume that pipelines divide a whole plant space into many small spaces. Spaces among the pipelines are defined by the shape of a rectangular solid. This small space is called a "partition space". Furthermore, the relation between adjacent partition spaces can be expressed by a graphical representation. By using this adjacent relation graph, the designer can draw any line from any point to any point. This paper deals with finding the shortest path between two points in the plant using available routes.

The shortest path problem has been solved on a computer by Dijkstra's method [6]. In [4], the shortest path of a piping route in a nuclear power plant was also solved by Dijkstra's method. In [5], the shortest path of a piping route on a ship's deck was solved by using dynamic programming, which was based on a visibility graph of deck space.

An optimal design system for piping routes based on a space model that includes the influence of pipelines on the space can be realized by using the power multiplication method.

In this paper, optimal design system means the following. It is almost impossible to automatically find the optimal piping route that satisfies all design conditions and requests. Originally, we did not intend to construct such a system, but instead tried to construct a system in which the designer could decide the optimal piping route by using information in addition to the shortest path offered by a computer, for example, the second and third shortest path.

In Section 2, we describe the total plant piping model for a CAD system and define the space model and its construction procedure. In Section 3, a space model for the shortest path problem of piping routes is discussed in detail. In Section 4, a solution method for the shortest path problem of a piping route is shown through examples, and distance correction is also mentioned. Conclusions are presented in Section 5.

2. TOTAL MODEL REPRESENTATION FOR A PLANT PIPING CAD SYSTEM

The total plant piping model can be expressed by the following graphical representation, which is based on computational geometry.

$$W = (P, E, V, F, V_A, V_B, V_C, V_D, V_a, V_b, V_c, V_d, V_e, V_f, V_g, V_h)\,, \tag{1}$$

where

P is the set of points of parts in the pipelines (Figure 2),
E is the set of pipeline segments (Figure 2),
V is the set of small spaces divided by the pipelines in plant space R,
F is the set of boundary surfaces of the partition spaces,
V_A, V_B, V_C, V_D are the relational functions between pipelines and partition spaces, and
$V_a, V_b, \ldots, V_h$ are the relational functions between pipe parts and partition spaces.

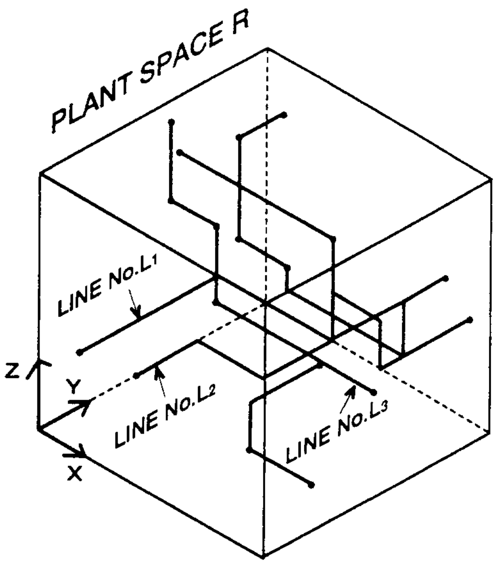


Figure 2. Plant space and pipeline model.

This model can completely express all information about all elements that exist in the plant except for equipment and building structures. This allows easy handling of the problem and a clear explanation of its structure. Such an abbreviation does not influence developing the logic of the space model definition.

The mathematical model expression such as equation (1) is famous in two-dimensional geography. We introduce it into the three-dimensional plant piping field and apply the model to actual problems of piping design, i.e., interference checking or the shortest path problem of pipelines.

Each item of equation (1) is explained in detail in the following section.

2.1. Graphical Representation of the Piping Model G_L

First, we explain the pipeline's mathematical model, which is the base of all mathematical models of plant piping. The power plant piping is constructed inside the building, and its final layout is shown in Figure 1. Actual plant data is very complex, and simplifying the data is needed to develop the logic. The plant used in the definition of the model is shown in Figure 2.

Inside the plant space R, building structures, equipment, and pipelines are constructed. The plant coordinate system and plant original point are settled in the plant space R. The positioning and orientation of all plant elements can make it possible by means of the coordinate system.

The graphical representation of pipeline G_L is defined as follows:

$$G_L = (P, E). \tag{2}$$

When it is only necessary to design pipelines, the parts of the piping are extracted from the total model W. Almost all conventional CAD systems are based on this model.

The pipeline installed between an outlet nozzle and an inlet is defined as one unit of pipeline and a line name is given for identification. In general, the route of a pipeline is designed from the center line of a pipe at the planning stage.

We use the center line of a pipe as the pipeline model. The pipeline represented by a center line is composed of the part's points and the line segments connecting the points. And the part's point are assumed to be composed of a terminal point, a bend point, and a branch point. By using this composition, the piping system can be expressed by the above graphical representation of equation (2). The part's point is referred to as the graph node (hereafter, called pipe node) and the line segment is referred to as the graph arc (hereafter, called pipe arc).

Details of equation (2) are as follows.

L_i is the i^{th} pipeline.
p_{ij} is the j^{th} pipe node expressed by the part's point on L_i and the coordinate is denoted by (x_{ij}, y_{ij}, z_{ij}).
e_{ik} is the k^{th} pipe arc of the line segment on L_i.

Then, each set of pipe nodes and arcs for all pipelines are shown as follows.

$$P = \{p_{ij} \mid j = 1, \dots, n_{pi},\ i = 1, \dots, n_l\}, \tag{3}$$

and

$$E = \{e_{ij} \mid k = 1, \dots, n_{ei},\ i = 1, \dots, n_l\}, \tag{4}$$

where

n_{pi} is the number of pipe nodes of L_i,
n_{ei} is the number of pipe arcs of L_i, and
n_l is the number of pipelines.

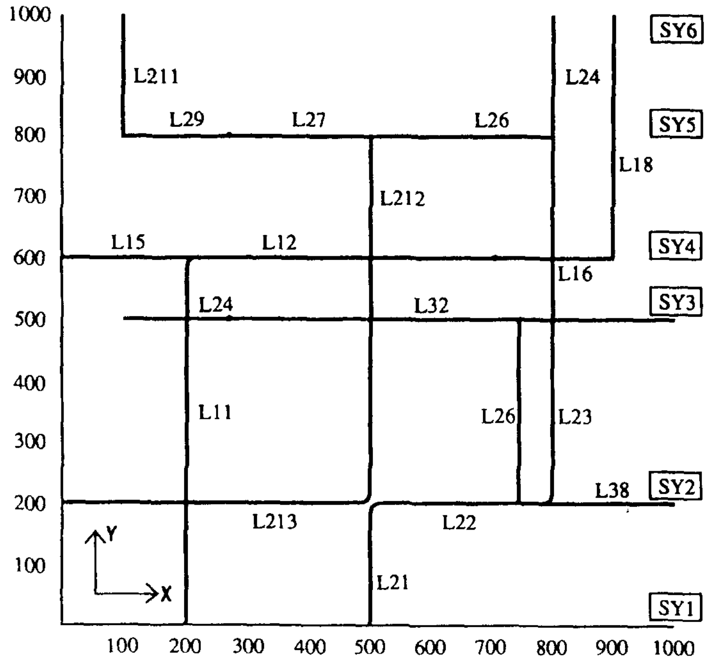
2.2. Graphical Representation of the Space Model G_V

The definition of a space model and its mathematical representation are described in this section, and the procedure for constructing the space model is explained.

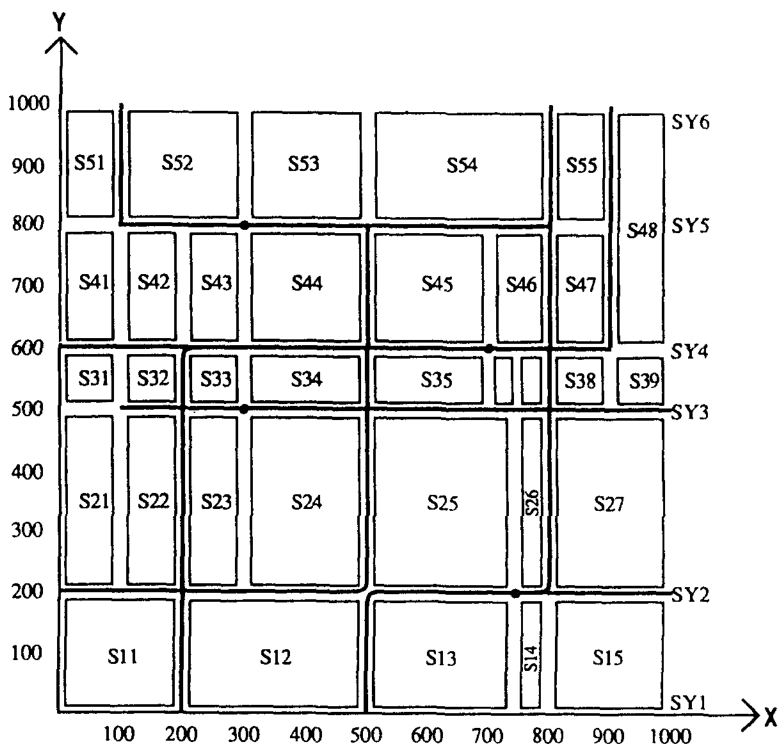
2.2.1. Definition of partition space V

A plant space R is divided into small spaces around pipeline segments. These spaces are considered to be divided by the pipeline arcs and nodes in order to satisfy the following conditions.

(1) The shape of a small space is a rectangular solid and the six surfaces are parallel to either the plane of X-Y, Y-Z, or Z-X of the plant coordinate system.
(2) There are no gaps and overlaps among the spaces.
(3) The plant space R is perfectly filled by these spaces.



(a) Plan drawing of Figure 2.

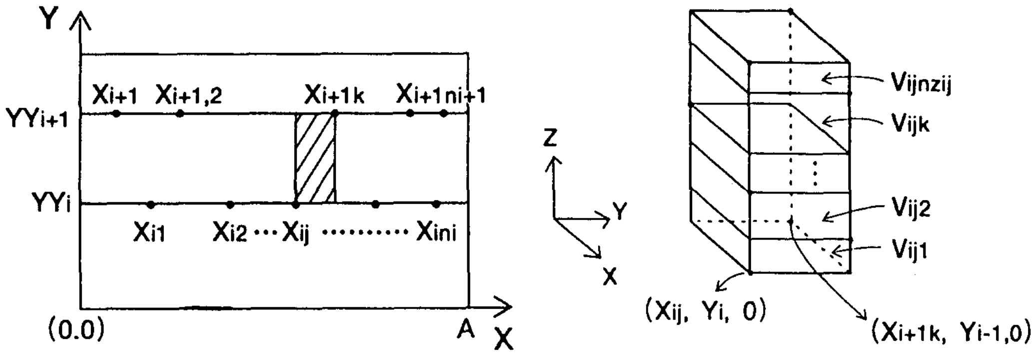


(b) Generation of rectangular.

Figure 3.

The small space disected from a plant space R by the following method is defined as the partition space. The shape of the partition space is rectangular solid (Figure 3).

Step 1 Draw the plan drawing of pipe lines in the plant space R and draw horizontal lines at the node points of the pipe lines (Figure 3a).



(c) One of rectangular. (d) Creation of rectangular solid.

Figure 3. (cont.).

Step 2 Generate rectangulars between two horizontal lines in the plan drawing by drawing vertical lines at the node points on the horizontal lines (Figure 3b).

Step 3 Build the quadrate columns on each rectangular and create the rectangular solids by cutting the column with the parallel planes to the X-Y plane passed through at the node points on the edges of the column (Figure 3d).

2.2.2. Mathematical representation of a space model

The above mentioned partition spaces are adjacent to each other and have adjacent relationships with other partition spaces in six directions of the plant coordinate system. Each partition space and its adjacent relations are expressed by the following graphical representation.

$$G_V = (P_V, E_V), \tag{5}$$

where

P_V is the set of nodes that represents the partition spaces, and

E_V is the set of edges that represents the adjacent relationship between two partition spaces. Hereafter, this edge is called the adjacent relation edge.

3. A SPACE MODEL FOR SHORTEST PATH PROBLEM

In plant piping CAD, the model of plant piping W expresses all information about piping and can be applied to any design. For example, it is possible not only to design a new pipeline, but also to check mutual interference among plant elements and to search a space for maintenance of the plant elements.

When systematization of piping CAD advances to automatic design of a piping route, it will be necessary to add physical meaning to the space model and to arrange it to satisfy various kinds of design conditions.

3.1. Arrangement of the Space Model for the Shortest Path Problem of a Pipeline

The space model mentioned in Section 2.2 is a general form. When the graph is applied to the shortest path problem of a pipeline, it must be arranged by considering the following conditions.

(1) If plant equipment or building structures exist in the layout, the partition space must be deleted from the general space model. Furthermore, the edge connected to the partition space must also be deleted.

(2) When a common rectangular surface to which two partition spaces are adjacent (hereafter, called an adjacent common face) does not have an area large enough for new piping to pass through the adjacent relation edge between the two partition spaces must be deleted.

The method of deleting and preserving a partition space will be explained in detail. However, before that explanation, the physical characteristics of the partition space for the shortest path problem are defined

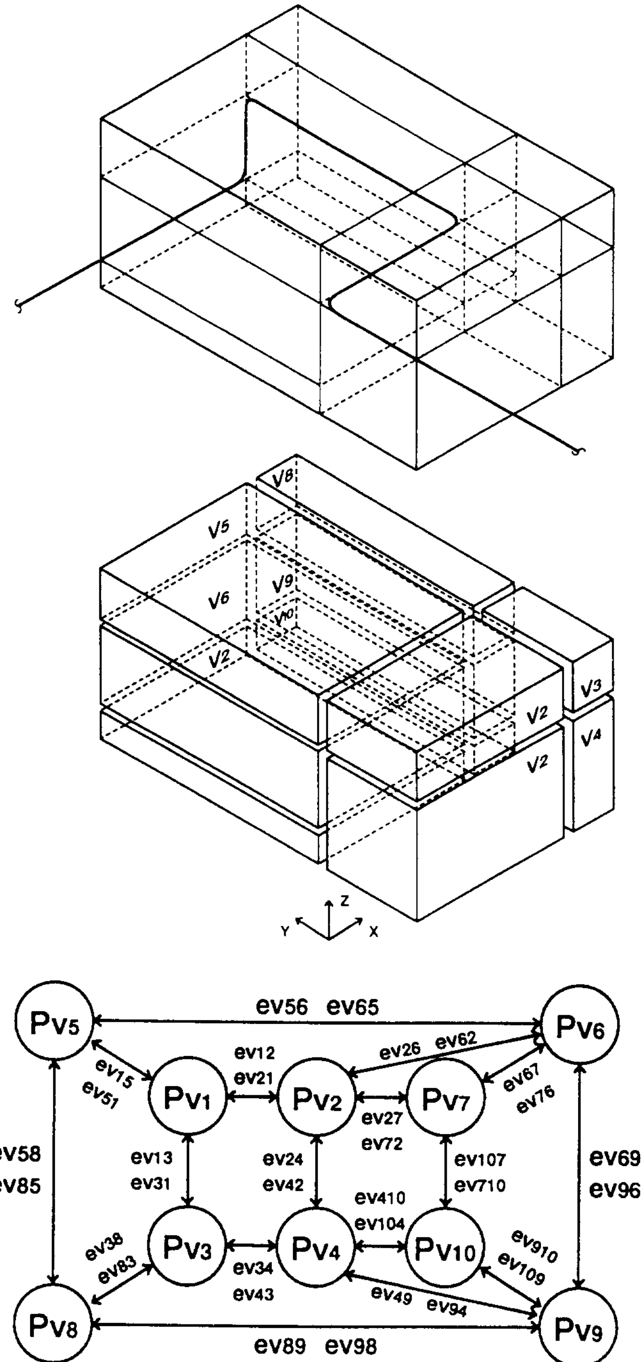


Figure 4. A sample of an adjacent relation graph.

3.1.1. Physical characteristics of the partition space

When the mathematical model is applied to an actual problem of piping design, particular physical meanings are given to the space model (5). That is to say, the piping route design must define the position of the partition space node in the plant space R; the pipeline node is assumed to pass through the position of the node. The position of the node is defined as the center of gravity of the partition space node.

The coordinate $({x^v}_i, {y^v}_i, {z^v}_i)$ of the position of the node p_{vi} is defined as follows:

$$({x^v}_i, {y^v}_i, {z^v}_i) = (x_{Gi}, y_{Gi}, z_{Gi}), \tag{6}$$

where (x_{Gi}, y_{Gi}, z_{Gi}) is the center of gravity of the partition space p_{vi}.

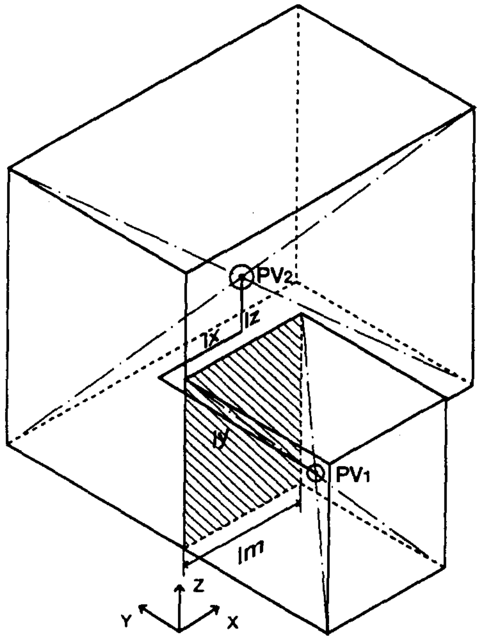


Figure 5. Distance between two partition space.

Next, the distance between adjacent partition spaces is defined by the following equation, which is essential to solve the shortest path problem (Figure 5). Let d_{ij} be the distance between two partition spaces, then

$$d_{ij} = |x^v{}_i - x^v{}_j| + |y^v{}_i - y^v{}_j| + |z^v{}_i - z^v{}_j| . \tag{7}$$

3.1.2. Deletion of a partition space node by equipment or building structures

In the above explanation, the partition space is divided from the plant space R by the pipelines. The influence of existing plant equipment or building structures on creating the space model was not considered.

Consideration of such influence is realized by deleting partition spaces from the space model in which the equipment or building structures exist.

When deciding whether deletion is necessary or not, the minimum rectangular solid that envelopes the equipment or building structure is considered. If overlapping between the rectangular solid and any partition space is known as a result of a Boolean operation between them, the partition space must be deleted from the original space model.

3.1.3. Deciding pipe diameter

The method of model construction was developed in Section 2.2.1, and it was assumed that an edge is generated between two adjacent partition spaces. However, the model is applied to the shortest path problem of a pipeline, it must be decided whether a new pipeline can pass through an adjacent common face or not

As mentioned above, if a pipeline passes through a partition space, the pipeline needs to pass through the center of gravity of the partition space. Thus, the adjacent common face must have an area large enough for the pipe. If the area is insufficient, the edge between two partition spaces should be removed from the graph.

At the same time, preventing interference with existing pipelines is considered. In order to make this decision, the following function is introduced.

$$F_d = \min \{t_d - r_{d\,\max} - r_p,\ t_1 - r_{1\,\max} - r_p,\ t_2 - r_{2\,\max} - r_p\} , \tag{8}$$

where

t_d is the half-length of the diagonal of an adjacent common face,
t_1 is the length of face's longer side,
t_2 is the length of face's shorter side,
$r_{d\,\max}$ is the maximum pipe radius of piping parts existing on face corner,
$r_{1\,\max}$ is the maximum pipe radius of piping existing on longer side of face,
$r_{2\,\max}$ is the maximum pipe radius of piping existing on shorter side of face, and
r_p is the pipe radius of newly installed piping.

The decision is made as follows.

If $F_d > 0$, then the partition space edge is set on the graph.
If $F_d \leq 0$, then the partition space edge is not set on the graph.

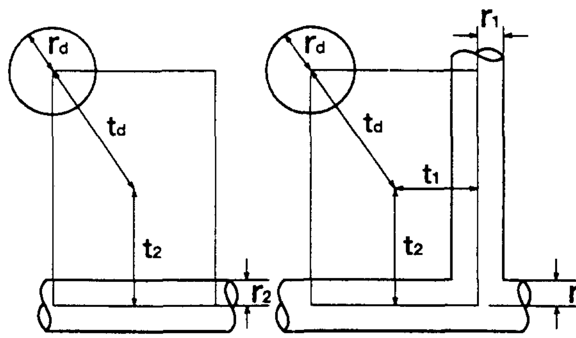


Figure 6. A pattern of adjacent common face.

One example of F_d is illustrated in Figure 6. In the left figure, $F_d = \min\{t_d - r_d - r_p,\ t_2 - r_2 - r_p,\ t_1 - r_p\}$. In the right figure, $F_d = \min\{t_d - r_d - r_p,\ t_1 - r_1 - r_p,\ t_2 - r_2 - r_p\}$.

4. SOLUTION OF THE SHORTEST PATH PROBLEM USING THE SPACE MODEL

The method to solve the shortest path problem for a pipeline is explained by using the space model mentioned above. We consider a problem where a pipeline is installed from an outlet nozzle to an inlet nozzle in a space where several pipelines already exist.

The conditions are as follows.

(1) The newly installed pipeline does not interfere with other pipelines.
(2) The total path length of the pipeline is as short as possible.
(3) When there exist more than two shortest paths, the path with the smallest number of bends is selected as the shortest path.

Hence, the main subject is to solve the problem by applying the power multiplication method to the distance matrix decided from the space model. This space model is the model arranged by the physical and design conditions mentioned above. The method is explained in the following section.

4.1. Solving the Piping Shortest Path Problem

Step 1 Create partition spaces within the range of the space by using existing pipelines, and include the outlet nozzle (start node) and inlet nozzle (finish node).
Step 2 At the same time, set the adjacent relation among the partition spaces.
Step 3 Calculate the value of the decision function for each adjacent common faces.

The above steps are done in the preparatory stage. The following are the actual execution steps for solving the shortest path problem.

Step 4 Delete the nodes and the arcs of the partition space according to the following conditions.

Step 4-1 Conditions based on the layout of plant equipment and building structure. Delete the partition space nodes and the edges from the original space model, that overlap with plant equipment and building structures.

Step 4-2 Physical condition of the pipe diameter. Delete the partition space edges from the original space model when the decision function is less than zero.

Step 5 Calculate the distance matrix between adjacent partition spaces from the final space model of a graphical representation. The position is the center of gravity of the partition space shown in equation (6); the distance between adjacent partition spaces is obtained from equation (7).

Step 6 Calculate the shortest path from the start node to the finish node by applying the power multiplication method to the distance matrix between adjacent partition spaces.

4.2. Adding Function to the Multiplication Method

For the purpose of the optimizing the piping route design system, some functions are added to the algorithm of the power multiplication method. This allows calculation of routes other than the shortest path.

(1) If two or more paths exist, the system can list all routes with same length as the shortest path.

(2) The system can calculate the second shortest path.

Function (1) was added to allow selection of path with the smallest number of bends. Function (2) was added because the shortest path is not necessarily optimal. There are many factors that allow optimizing the piping route, and deciding the optimal is left to the designer. Therefore, the system must offer the designer maximum information related to the shortest path. The shortest path is $p_{vs} \rightarrow p_{vj} \rightarrow p_{ve}$ because of fewer bends.

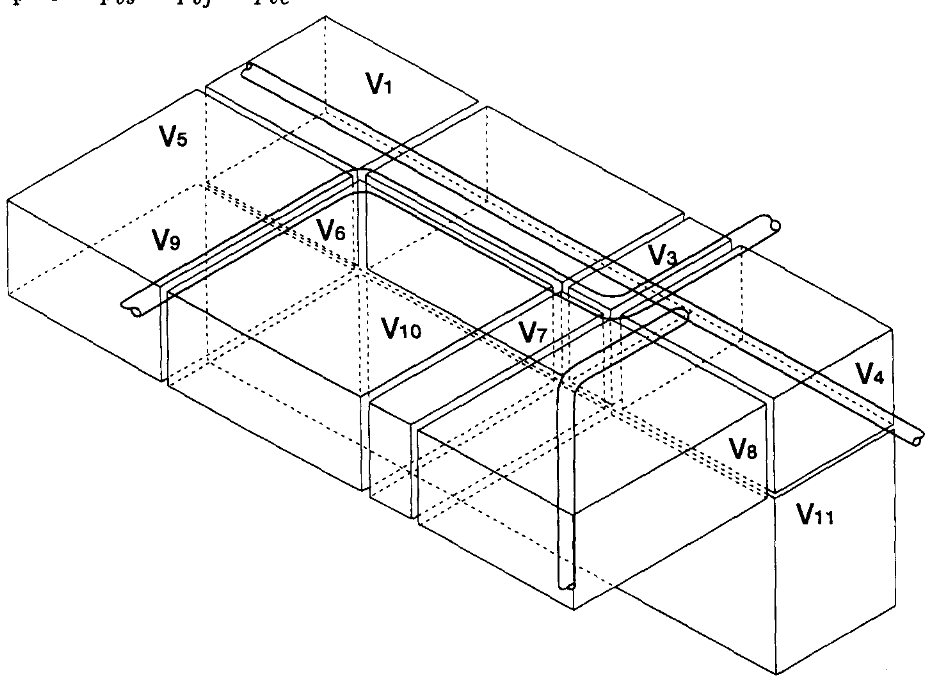


Figure 7. Sample model for shortest path problem.

4.3. Shortest Path Problem

We will explain the application of the above method to a case in which a pipeline is installed among already existing pipelines that is shown in Figure 7. The point S in the partition space p_{v1} denotes the start point of the new pipeline L_0, and the point in the partition space p_{v4} denotes

Table 1. Space model of Figure 7.

$G_V=\{P_V,E_V\}$	
P_V	$\{P_{V1},P_{V2},P_{V3},P_{V4},P_{V5},P_{V6},P_{V7},P_{V8},P_{V9},P_{V10}$ $P_{v11}\}$
E_V	$\{e_{v12},e_{v21},e_{v19},e_{v91},e_{v15},e_{v51},e_{v23},e_{v32},e_{v210},e_{v102}$ $e_{v26},e_{v62},e_{v34},e_{v43},e_{v37},e_{v73},e_{v311},e_{v113},e_{v47}$ $e_{v74},e_{v411},e_{v114}\}$

Table 2. The pattern types of common faces and function values.

Part. Space (Distance)	Func. Val.	Part. Space (Distance)	Func. Val.	Part. Space (Distance)	Func. Val.
$V_1 - V_2$ (225)	30	$V_2 - V_3$ (150)	10	$V_3 - V_4$ (125)	10
$V_1 - V_5$ (200)	−72	$V_2 - V_6$ (200)	10	$V_3 - V_7$ (200)	10
$V_4 - V_8$ (200)	72	$V_1 - V_9$ (150)	15	$V_2 - V_{10}$ (150)	15
$V_3 - V_{11}$ (100)	−25	$V_4 - V_{11}$ (150)	15	$V_5 - V_6$ (225)	10
$V_6 - V_7$ (150)	95	$V_7 - V_8$ (125)	95		

the finish point. Two pipelines, L_1 and L_2, already exist, whose diameter is 20 cm and 60 cm, respectively.

Then, the problem is solved about the case where the diameter of L_0 is 60 cm. The solution will be explained according to the procedure described in Section 4.1.

Step 1 The space shown in Figure 7 is divided into 11 partition spaces, $p_{vi}, \ldots, p_{v11}$ by L_1 and L_2.

Step 2 The space model of Figure 7 is shown in Table 1.

Step 3 The value of the decision function at each common face between two adjacent partition spaces are shown in Table 2.

Step 4 In Figure 7, there is neither equipment nor building structures, therefore, all 11 partition spaces are not deleted from the original space model.

Step 5 The distance between two adjacent partition spaces and the calculated value of the decision function is shown in Table 2.

Step 6 The distance matrix for L_0 = 60 cm is shown below.

$$\begin{pmatrix} 0 & 225 & \infty & \infty & 200 & \infty & \infty & \infty & 150 & \infty & \infty \\ 225 & 0 & \infty & \infty & \infty & 200 & \infty & \infty & \infty & 150 & \infty \\ \infty & \infty & 0 & 125 & \infty & \infty & 200 & \infty & \infty & \infty & \infty \\ \infty & \infty & 125 & 0 & \infty & \infty & \infty & 200 & \infty & \infty & 175 \\ 200 & \infty & \infty & \infty & 0 & 225 & \infty & \infty & \infty & \infty & \infty \\ \infty & 200 & \infty & \infty & 225 & 0 & 150 & \infty & \infty & \infty & \infty \\ \infty & \infty & 200 & \infty & \infty & 150 & 0 & 125 & \infty & \infty & \infty \\ \infty & \infty & \infty & 200 & \infty & \infty & 125 & 0 & \infty & \infty & \infty \\ 150 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 225 & \infty \\ \infty & 150 & \infty & \infty & \infty & \infty & \infty & \infty & 225 & 0 & 250 \\ \infty & \infty & \infty & 175 & \infty & \infty & \infty & \infty & \infty & 250 & 0 \end{pmatrix}$$

STEP 7. The shortest path problem is solved based on the above matrices by means of the power multiplication method. The solution is as follows.

- Results of calculation by the power multiplication method are shown in Table 3.

- Shortest path: No. 1: $p_{v1} \to p_{v2} \to p_{v10} \to p_{v11} \to p_{v4}$. No. 2: $p_{v1} \to p_{v9} \to p_{v10} \to p_{v11} \to p_{v4}$.
- Number of bends: No. 1 is three and No. 2 is two.

Therefore, the shortest path is the No. 2 route. The total path length of the shortest path is 800 cm. The second shortest path is $p_{v1} \to p_{v5} \to p_{v6} \to p_{v7} \to p_{v8} \to p_{v4}$ and the total path length of the second shortest path is 900 cm.

Table 3. Calculation results of the distance matrix.

No.	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}	p_{18}	p_{19}	p_{110}	p_{111}
(1)	225 1 2	∞	∞	200 1 5	∞	∞	∞	150 1 9	∞	∞
(2)	225	∞	∞	200 2 6 5 6	425	∞	∞	150 2 10 9 10	375	∞
(3)	225	∞	∞	200	425 6 7	575	∞	150	375 1011	625
(4)	225	∞	800 114	200	425	575	700 7 8	150	375	625
(5)	225 7 3	775 (900) 8 4	800	200	425	575	700	159	375	625
(6)	225	775	800	200	425	575	700	150	375	62

Note 1: p_{1j} denotes the shortest path length from point 1 to i.

Note 2: The second line and third line in each frame denote the previous node number to the current node. The third line means that there are more than two paths whose lengths are the same. For example, in frame 2, there exist two paths for the shortest path to node 10; one from node 2 and one from node 9.

Note 3: The description (900) shown in frame 5 denotes the second shortest path to node 4 from node 1.

5. CONCLUSIONS

In consideration of the above described theory, the following two points will be mentioned.

(1) Effectiveness of the power multiplication method for this purpose.
(2) Practicability of a CAD system installed automatically varies methods for solving distance matrices which have been proposed and used in practice.

The reasons for selecting the power multiplication method are as follows.

(1) The algorithms are very simple and trial and error is not necessary.
(2) Matrix calculation processing is very clear and function addition is easy.
(3) Not only the shortest, but also the second and third shortest paths between two points can be obtained.

As for the practicability of using a CAD system, it is not considered that automatic design of all plant piping can be initially accomplished using this automation technique. It is not intended to say that this theory obtains the optimal shortest path design for all plant pipelines. It is also fair to say that this is the only deficiency of this report.

This theory is sufficiently practical and is effective for the optimal shortest path problem over a partial range. When piping routes have been designed to some degree using a piping CAD system and the pipelines are to be filled into complicated spaces, efficiency and accuracy will be improved by carrying out space division over the required range, preparing adjacent graphs and obtaining solutions by the power multiplication method. It is shown that the automation system based on this theory is very effective.

By pursuing compactness of the automation system, many practical applications will be found. Apart from discussing the system, we want to conclude this consideration with the understanding that propriety and practicability of this theory have been fully shown.

REFERENCES

1. M. Iri and T. Koshizuka, *Computational Geometry and Geographic Information* (in Japanese), Kyouritsu Shuppan, (1993).
2. T. Asano, *Computational Geometry* (in Japanese), Asakura Shoten, (1993).
3. S. Kund, An $O(n)$ algorithm for determining the subregion-tree representation of a rectangular dissection, *SIAM J. Comp.* **22** (1), 79–101 (1993).
4. G.E. Wangdahl, S.M. Pollock and J.B. Woodward, Minimum trajectory pipe routing, *Journal of Ship Research* **18** (1), 46–49 (1974).
5. T. Yoshinaga, T. Kiguchi, S. Tokumatu and K. Kumamoto, Three dimensional computer aided design system for plant layout (in Japanese), *Hitachi Hyouron* **68** (4), 59–64 (1993).
6. E.W. Dijkstra, A note on two problems in connexion with graph, *Numerische Mathematik* **1**, 269–271 (1959).
7. B. Carre, *Graphs and Networks*, Clarendon Press, Oxford, (1979).
8. A.V. Aho, J.E. Hopcroft and J.D. Ullman, *The Design and Analysis of Computer Algorithms*, Addison-Wesley, Reading, MA, (1974).